Section 5.2 – Verifying Trig Identities!

We are going to use the same process as in Section 5.1, but now we have an equation instead of an expression. Here, we want to PROVE or VERIFY that the left side of the equation equals the right side.

Recall: **ALL** of the identities we learned:

$$\sin u=\frac{1}{\csc u} \quad \cos u=\frac{1}{\sec u} \quad \tan u=\frac{1}{\cot u}$$
$$\csc u=\frac{1}{\sin u} \quad \sec u=\frac{1}{\cos u} \quad \cot u=\frac{1}{\tan u}$$

$$\tan u=\frac{\sin u}{\cos u} \quad \cot u=\frac{\cos u}{\sin u}$$

$$\sin^2 u+\cos^2 u=1 \quad 1+\tan^2 u=\sec^2 u \quad 1+\cot^2 u=\csc^2 u$$

$$\sin(-x)=-\sin x \quad \cos(-x)=\cos x \quad \tan(-x)=-\tan x$$
$$\csc(-x)=-\csc x \quad \sec(-x)=\sec x \quad \cot(-x)=-\cot x$$

$$\sin\left(\frac{\pi}{2}-u\right)=\cos u \quad \cos\left(\frac{\pi}{2}-u\right)=\sin u$$
$$\tan\left(\frac{\pi}{2}-u\right)=\cot u \quad \cot\left(\frac{\pi}{2}-u\right)=\tan u$$
$$\sec\left(\frac{\pi}{2}-u\right)=\csc u \quad \csc\left(\frac{\pi}{2}-u\right)=\sec u$$

TIPS....

- Work with one side of the equation at a time (start with the more complicated side)
- USE THE IDENTITIES!
- If having trouble...get in terms of sines and cosines

EXAMPLES: Verify the identity

1. $\dfrac{\sec^2\vartheta-1}{\sec^2\vartheta}=\sin^2\vartheta$

$\dfrac{\tan^2\vartheta}{\sec^2\vartheta}=\dfrac{\sin^2\vartheta}{\cos^2\vartheta}\cdot\dfrac{\cos^2\vartheta}{1}$

$=\sin^2\vartheta$ ✓

2. $\dfrac{1}{1-\sin\alpha}+\dfrac{1}{1+\cos\left(\frac{\pi}{2}-\alpha\right)}=2\sec^2\alpha$

$=\dfrac{1}{1-\sin\alpha}+\dfrac{1}{1+\sin\alpha}$

$=\dfrac{(1+\sin\alpha)+(1-\sin\alpha)}{(1-\sin^2\alpha)}$

$=\dfrac{2}{\cos^2\alpha}=2\sec^2\alpha$ ✓

Section 5.2 – Verifying Trig Identities!

3. $(\tan^2 x+1)(\cos^2 x-1) = -\tan^2 x$

$= \sec^2 x(-\sin^2 x)$

$= \frac{1}{\cos^2 x} \cdot \frac{-\sin^2 x}{1} = -\tan^2 x$ ✓

4. $\cot\left(\frac{\pi}{2}-x\right)-\cot(-x) = \sec x \csc x$

$= (\tan x)+\cot x$

$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$= \frac{1}{\sin x \cos x} = \sec x \csc x$ ✓

5. $\sec(-y)+\tan y = \frac{\cos y}{1-\sin y}$

$= \sec y + \tan y$

$= \frac{1}{\cos y} + \frac{\sin y}{\cos y}$

$= \frac{1+\sin y}{\cos y}\left(\frac{1-\sin y}{1-\sin y}\right)$

$= \frac{1-\sin^2 y}{\cos y(1-\sin y)}$

$= \frac{\cos^2 y}{\cos y(1-\sin y)}$

$= \frac{\cos y}{1-\sin y}$ ✓

6. $\frac{\cot^2 \vartheta}{1+\csc \vartheta} = \frac{1-\sin \vartheta}{\sin \vartheta}$

$\frac{\csc^2 \vartheta - 1}{1+\csc \vartheta} = \frac{(\csc \vartheta - 1)(\csc \vartheta + 1)}{(\csc \vartheta + 1)}$

$= \frac{1}{\sin \vartheta} - 1$

$= \frac{1-\sin \vartheta}{\sin \vartheta}$ ✓

HW: p.387: #1, 5, 10, 13, 23, 27, 29